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general parasitism to a highly specific parasitism, in the latter cases the parasite having become so perfectly modified to fit itself to the one species of host that it must mean an advanced stage of evolution.

In considering the importation of parasite species we must aim to have both kinds if they exist—the specifically constituted parasites and the general parasites. From the ranks of the latter we will have great help in the destruction of large numbers of the particular insect against which it was imported, but it will have the additional advantage of accommodating itself to other injurious species, and hence on the whole will probably be a more desirable addition to the American parasite fauna.

Although not parasites, mention should be made of two species of ground-beetles brought over from Europe in the course of the gipsy moth work. *Calosoma sycophanta* has been found to attack a number of injurious native caterpillars; and *Carabus auratus*, while probably primarily a feeder upon slugs, as it is in its native home, also feeds upon cutworms and probably other soft-bodied insects ordinarily found in or near the ground.

ON THE LOCATION OF THE ROOTS OF THE DERIVATIVE OF A POLYNOMIAL

BY J. L. WALSH

DEPARTMENT OF MATHEMATICS, HARVARD UNIVERSITY

Communicated April 11, 1922

This note considers some geometric aspects of the problem of the approximate location of the roots of the derivative $f'(z)$ of a polynomial $f(z)$ when the roots of $f(z)$ are known. If $f(z)$ has m_1 roots in a circle C_1, m_2 roots in a circle C_2, and no other roots, the roots of $f'(z)$ are known to lie in C_1, C_2, and a certain third circle C which is readily determined.[1] Moreover, if $f(z)$ has m_1 roots in C_1, m_2 roots in C_2, m_3 roots in C_3, and no other roots, and if the circles C_1, C_2, C_3, are equal and have collinear centers, the roots of $f'(z)$ are known to lie in those circles and in two circles C', C'' equal to the original circles and whose centers are collinear with their centers.[2] In each of these cases, the actual *locus* of the roots of $f'(z)$ consists of the circles stated, when the given circles are the loci of those roots of $f(z)$ (supposed to vary independently) which they contain.

The second of these results suggests the problem of the determination of the locus of the roots of $f'(z)$ when the locus of the roots of $f(z)$ consists of three circles C_1, C_2, C_3 which are not supposed equal and with collinear centers.[3] The solution of that problem is indicated in the present note; the answer is stated in the

THEOREM. *Let circles C_1, C_2, C_3 be the respective loci of m_1, m_2, m_3 roots of a polynomial $f(z)$. Then the locus of the roots of $f'(z)$ consists of C_1, C_2,*

C_3 (except that C_i is to be omitted if m_i = 1) and of a region R bounded by the bicircular quartic

$$\begin{array}{ll}
 & m_1{}^2[(x-b_1)^2 + (y-b_2)^2-r_2{}^2]\ [(x-c_1)^2 + (y-c_2)^2-r_3{}^2] \\
 & +m_2{}^2[(x-a_1)^2 + (y-a_2)^2-r_1{}^2]\ [(x-c_1)^2 + (y-c_2)^2-r_3{}^2] \quad \times \\
 & +m_3{}^2[(x-a_1)^2 + (y-a_2)^2-r_1{}^2]\ [(x-b_1)^2 + (y-b_2)^2-r_2{}^2] \\
(1) & +2m_2m_3[(x-a_1)^2 + (y-a_2)^2-r_1{}^2]\ [(x-b_1)\ (x-c_1) + (y-b_2(\ (y-c_2)-r_2r_3] \\
 & +2m_1m_3[(x-b_1)^2 + (y-b_2)^2-r_2{}^2]\ [(x-a_1)\ (x-c_1) + (y-a_2)\ (y-c_2)-r_1r_3] \\
 & 2m_1m_2[(x-c_1)^2 + (y-c_2)^2-r_3{}^2]\ [(x-a_1)(x-b_1) + (y-a_2)\ (y-b_2)-r_1r_2] = 0;
\end{array}$$

here the circles C_1, C_2, C_3 have the respective centers and radii: $(a_1, a_2), r^1$; (b_1, b_2), r_2; (c_1, c_2), r_3. If the curve (1) consists of two nested ovals, R is the region bounded by the outer oval; if (1) does not consist of two nested ovals, R is the interior of the curve.

If no two of the regions C_1, C_2, C_3, R have any point in common, they contain respectively $m_1 - 1$, $m_2 - 1$, $m_3 - 1$, 2 roots of $f'(z)$. If R is in two parts, which have no point in common with each other or any of the regions C_i which is a part of the locus of the roots of the derivative, each part of R contains one root of $f'(z)$.

This theorem replaces the problem of the exact location of the roots of $f'(z)$, which depends on the solution of an equation of degree $m_1 + m_2 + m_3 - 1$, by the problem of their approximation which is solved by plotting a quartic curve, which depends on the solution of equations of degree 4. In particular cases—such as that mentioned in the first paragraph of this note—the quartic (1) degenerates, in which case the new problem is still simpler, for it depends merely on the solution of quadratics.

The theorem can be proved by a straightforward method which we now indicate. The locus of the roots of $f'(z)$ is readily found when C_1 is not a null-circle but C_2 and C_3 are both null-circles; every point of the boundary is a point of the curve (1) where $r_2 = r_3 = 0$. The locus of the roots of $f'(z)$ can then be found when neither C_1 nor C_2 is a null-circle by determining the envelope of the former boundary; every point of the new boundary is a point of (1) where $r_3 = 0$. Finally the locus is found in the general case by determining the envelope of this last determined boundary; every point of the boundary is a point of the curve (1).

A particularly interesting case of the theorem occurs if $m_1 = m_2 = m_3$, $r_1 = r_2 = r_3$, and the centers of C_1, C_2, C_3 are the vertices of an equilateral triangle. The locus of the roots of $f'(z)$ is (in addition to the regions C_i) the interior and boundary of a circle whose center is the center of this triangle and whose radius is $(r_1^2 + r_1h)^{1/2}$, h being the distance from the center of the triangle to the centers of the circles C_i.

The geometry of this entire situation deserves further inquiry, with especial reference to the cases where (1) degenerates. Our theorem generalizes presumably to the case in which any number of circles C_i form the loci of the roots of $f(z)$, and also where we consider not merely the first but the k^{th} derivative of $f(z)$. The writer hopes to continue the study of these problems.

[1] See paper by the writer, *Trans. Amer. Math. Soc.*, **22**, 1921 (101–116).

[2] See a forthcoming paper by the writer in the *Trans. Amer. Math. Soc.;* a similar result holds for any number of equal circles C_i which have collinear centers.

[3] This problem is set forth explicitly in the paper referred to in the preceding footnote.

ON THE DISTRIBUTION OF THE VELOCITIES OF STARS OF LATE TYPES OF SPECTRUM

BY GUSTAF STRÖMBERG

MOUNT WILSON OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Read before the Academy April 25, 1922

The recent determination of spectroscopic paralaxes or stars of the later types of spectrum made by Adams and his collaborators[1] at the Mt. Wilson Observatory has made it possible to compute the absolute motions in space of about 1300 stars of spectral types A_7 to M for which the proper motions and radial velocities are known. The spectroscopic parallaxes for which the errors are proportional to the parallaxes themselves are in general better adapted for this purpose than the trigonometric parallaxes since the accuracy of the smaller parallaxes is considerably greater.

The three velocity-components of these stars have been computed with reference to the galactic system of coördinates, the pole of galaxy being assumed to have the position $A = 190.6°$; $D = + 27.2°$. This system of coördinates is very convenient in the study of the systematic motions of the stars since these motions are mainly in the galactic plane.

Solar Motion.—The solar motion can be obtained directly by forming the algebraic mean of the velocity-components which are measured with reference to the sun. By taking the opposite vector we find the sun's velocity relative to the group of stars employed. The following values for the sun's velocity have been derived in this way.

SPECIAL TYPE	M	NO.	A_0	D_0	V KM.
A6 to F9..............	<3	153	269°	+27°	21.2
	>3	133	274	35	23.4
G0 to G9..............	<3	224	266	32	19.9
	>3	154	279	23	40.2
K0 to K9..............	<4	325	282	43	18.3
	>4	112	286	31	33.1
M....................	<4	98	271	44	16.2
A6 to M..............	Giants	800	273	37	18.8
A6 to M..............	Dwarfs	415	281	29	31.7